Homework 6

# 6.1 Donut Inference I

### Castle Point Bakery (CPB) makes delicious donuts. The high-tech production line is set up to make donuts with mean mass μ = 100 grams and standard deviation σ = 4 grams. Assume this information is accurate and donut mass follows a normal distribution. Compute the following for a single (N = 1) donut:

### A PDF plot for X, the mass of a CPB donut.

Solution:

### The 5th percentile CPB donut mass.

Solution:

N = 1

### The 95th percentile CPB donut mass.

Solution:

= 106.57

### The probability a CPB donut mass is ≤ 90 grams.

Solution:

= 0.006

### The probability a CPB donut mass is ≥ 110 grams.

Solution:

= 0.0062

# 6.2 Donut Inference II

### Hoboken-Os (H-Os) also produces delicious donuts. After N = 100 visits you have collected sample data showing the average donut mass to be ȳ = 99.1 grams. Assume H-Os has the same standard deviation as CPB (σ = 4 grams). Based on this data, compute the following:

### A PDF plot for Ȳ, the mean mass for N = 100 H-Os donut samples.

Solution:

### The 5th percentile mean mass for N = 100 H-Os donut samples.

Solution:

= 98.44

### The 95th percentile mean mass for N = 100 H-Os donut samples.

Solution:

= 99.76

### The probability the mean mass for N = 100 H-Os donut samples is ≤ 100 grams.

Solution:

= 0.9877

### A 95% confidence interval for the mean mass of H-Os donuts.

Solution:

Z0.025 = -1.96

Z0.975 = 1.96

Considering

= 99.1 ± (1.96 x 0.4)

= 99.1 ± 0.783

= [98.31, 99.88]

Considering 100 samples, the population mean will fall in the range [98.31, 99.88] 95% of the time.

# 6.3 Spring Break Recovery

### The binary random variable X measures the event of rolling a sum of 7 or 11 from a pair of dice (1 for rolling 7 or 11; 0 for anything else).

### Using a random dice generator, collect at least N = 30 samples for X.

Sample Space

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Sample Space | |  | SUM |  | Binary Equ |
| 1 | 5 |  | 6 |  | 0 |
| 4 | 5 |  | 9 |  | 0 |
| 5 | 1 |  | 6 |  | 0 |
| 3 | 3 |  | 6 |  | 0 |
| 3 | 1 |  | 4 |  | 0 |
| 3 | 5 |  | 8 |  | 0 |
| 2 | 5 |  | 7 |  | 1 |
| 3 | 2 |  | 5 |  | 0 |
| 1 | 5 |  | 6 |  | 0 |
| 6 | 4 |  | 10 |  | 0 |
| 6 | 5 |  | 11 |  | 1 |
| 1 | 1 |  | 2 |  | 0 |
| 1 | 4 |  | 5 |  | 0 |
| 2 | 6 |  | 8 |  | 0 |
| 4 | 2 |  | 6 |  | 0 |
| 2 | 2 |  | 4 |  | 0 |
| 4 | 2 |  | 6 |  | 0 |
| 2 | 6 |  | 8 |  | 0 |
| 4 | 3 |  | 7 |  | 1 |
| 4 | 4 |  | 8 |  | 0 |
| 4 | 2 |  | 6 |  | 0 |
| 2 | 4 |  | 6 |  | 0 |
| 4 | 4 |  | 8 |  | 0 |
| 5 | 4 |  | 9 |  | 0 |
| 3 | 6 |  | 9 |  | 0 |
| 1 | 1 |  | 2 |  | 0 |
| 5 | 6 |  | 11 |  | 1 |
| 2 | 4 |  | 6 |  | 0 |
| 3 | 3 |  | 6 |  | 0 |
| 6 | 2 |  | 8 |  | 0 |

### Compute the sample mean x̅.

Solution:

Mean x̅ = 0.13̇̇

### Compute the sample standard deviation sx and use it as an estimate of σ.

Solution:

Standard Deviation [Sample] = 0.345

[Calculation in Excel]

= 0.3399

### Compute the 95% confidence interval for the population mean, i.e. the true probability of rolling a 7 or 11.

Solution:

Z0.025 = -1.96

Z0.975 = 1.96

Considering

= 0.133 ± (1.96 x 0.062)

= 0.133 ± 0.121

= [0.011, 0.254]

Considering 100 samples, the population mean will fall in the range [0.011, 0.254] 95% of the time.

### Estimate how many samples would be required to reduce the 95% confidence interval to a maximum error |x̅ - μ| = 0.01.

Solution:

= 147.67 ≈ 148 samples

### Was the true probability of rolling a 7 or 11 within your 95% confidence interval? How often do “mistakes" happen?

Solution:

No, the probability didn’t fall under the 95% confidence interval. That may be due to the number of samples recorded for this experiment. I got a minimum error of 25% in 3 tries for this experiment.